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Contextuality and the Kochen-Specker Theorem

(Interpretations of Quantum Mechanics)

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Contextuality and the Kochen-Specker Theorem

ABSTRACT

Since local hidden variables are forbidden by Bell's theorem, the Kochen-Specker theorem forces a theory for quantum mechanics based on hidden variables to be contextual. Several experiments have been performed to proof this powerful theorem. I will review the consequences of the Kochen-Specker theorem and contextuality on the interpretation of quantum mechanics.

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INTRODUCTION

Beside the common Copenhagen interpretation of quantum mechanics there are hidden-variable theories postulated as a possible consequence of the EPR-Paradox. Most of them can be eliminated by so called "no-go theorems" for hidden variables. The most famous of them is the Bell's Theorem. The Kochen-Specker theorem complements the Bell's Inequality and excludes contextuality (in addition to locality) from hidden-variable theories too. Like Bell's theorem the Kochen-Specker theorem had been experimentally examined and these tests showed a perfect agreement with quantum mechanics and Copenhagen interpretation.

INTERPRETATIONS OF QUANTUM MECHANICS

Copenhagen Interpretation[iii]

The Copenhagen interpretation is designated after the Danish capital, where Niels Bohr and Werner Heisenberg were exchanging their thoughts about the probabilistic interpretation of wave function. Although there is no clear statement about this interpretation of quantum mechanics the heart of this idea is that the wavefunction has no reality. Nature is only probabilistic and only a measurement forces it to choose a state, before this there is no realism. The process of measurement causes a collapse of the wave function (this feature isn't included in all forms of the Copenhagen interpretation) and the result corresponds with the eigenvalue of the measurement operator. This is very important: you should mix up the operator with a real value. Furthermore Heisenberg's Uncertainty Principle prevents us of knowing all parameters of a system at once. One famous consequence of this interpretation is Schrödinger's Cat. Nowadays most physicists prefer the Copenhagen interpretation of quantum mechanics.

Hidden-Variable Theories

Due to the fact that the Copenhagen interpretation lacks realism and claims that the quantum world is only based on pure probability and statistics some famous scientists like Einstein stated that quantum mechanics is incomplete and that there must be a deeper reality.

Additional parameters have been introduced to make quantum mechanics deterministic. No-go-Theorems like Bell's or Kochen-Specker's in combination with experiments have cut down the features of possible hidden-variable theories. Locality and non-contextuality are forbidden for these theories, so that there have to be an interaction faster light speed. A famous hidden-variable theory is Bohmian mechanics[iii], which is non-local and contextual.

Other Interpretations

There are many more interpretations of quantum mechanics beside those two I've already mentioned above. The Many-worlds interpretation tries to save realism without hidden variables. The key statement of this interpretation is that every possibility becomes true, but in a different reality. The universe itself constantly splits into almost infinite slightly different worlds and in each of them one possible outcome of a quantum process becomes true. This theory implies many interesting aspects like quantum immortality and numerous parallel universes. Despite the almost impossibility to prove this theory it shows a couple of advantages like an explanation for a fine-tuned universe (anthropic principle) or the removal of an observer dependence in quantum mechanics. Other interpretations like the "Many-minds" or "consciousness cause collapse" (CCC) are introducing unnecessary assumptions about human minds into quantum mechanics. In addition to them there are interpretations that only slightly differentiate from those I've already mentioned.

Comparison[iv]

The different interpretations are characterised by several features like realism, completeness, localism and determinism.

Interpretation	**Realism**	**Localism**	**Determinism**	**Unique History**	**Observer conscience**
Copenhagen	No	Yes	No	Yes	No
Hidden Variables	Yes	No	Yes	Yes	No
Many Worlds	Yes	Yes	Yes	No	No
Many Minds	Yes	Yes	Yes	No	Yes
CCC	Yes	Yes	No	Yes	Yes

Tab.1: a comparison of the different interpretations of Quantum Mechanics

BELL’S THEOREM

EPR-Paradox[v]

The EPR-Paradox is a Gedankenexperiment of Albert Einstein, Boris Podolsky and Nathan Rosen, that intended to show the incompleteness of quantum mechanics. By adding conditions like realism, locality, completeness and counterfactual definiteness to quantum mechanics you will have to expand your theory with hidden variables to explain effects like entanglement (Einstein called it: “spooky action at distance”) because else all this seemingly reasonable conditions together cause a contradiction. Another possibility to solve this problem is to give up realism and counterfactual definiteness by augmenting with the Copenhagen interpretation. Furthermore Bell[vi] showed that even with hidden variables you can’t keep all this conditions together, because you will have to give up locality to stay in agreement with the experiments. The experiment for the EPR-Paradox in Bohm’s formulation can be imagined like that: A source emits two electrons that have been entangled (prepared to occupy a spin singlet in this case). Now there are two possible states for the electrons concerning the spin along the z-axis, either electron A has spin up and electron B has spin down or the other way round. The electron A is emitted into direction of an observer called Alice and the electron B will be sent to Bob who sits in the opposite direction of Alice seen from the source.

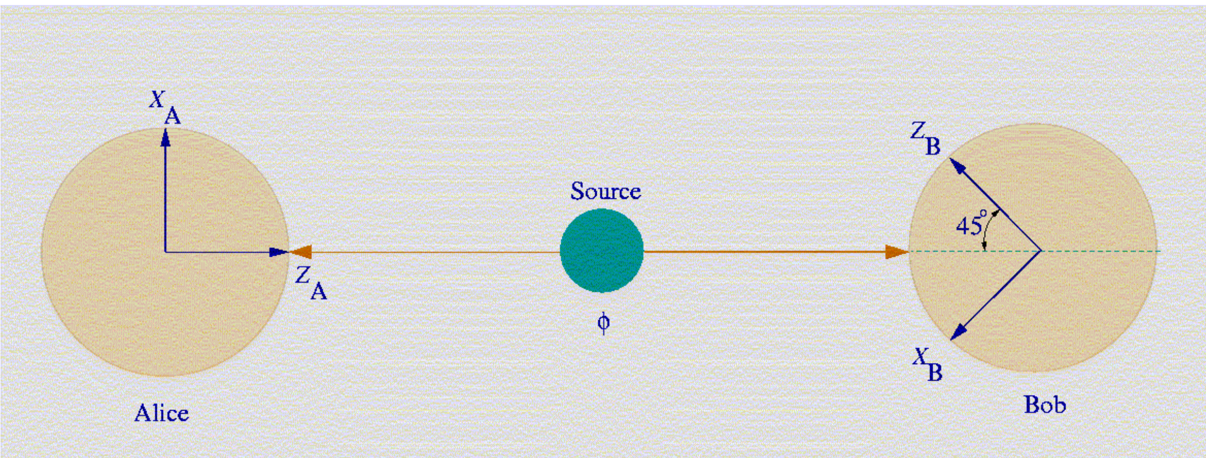

Fig.1: entangled electrons are sent to Alice and Bob, who perform spin measurements

When Alice and Bob measure the spin along the z-axis, they will get the values I’ve described above. But if Bob decides to measure the x-spin he has a 50-50 probability to get spin up (or spin down). The Heisenberg uncertainty principle tells us that it is impossible to know the values of two non-commuting variables (like the components of the spin) at the same time, but due to the entanglement of the z-spin you will expect that you can also measure the z-spin of the electron that has been measured by Bob simply from Alice' measurement. This doesn't work and the outcome of Alice' measurement of the z-component of the spin isn't any longer predictable.

Inequality[vii][viii]

If there were hidden variables the correlation between Alice's and Bob's results would be different than you would expect from pure quantum mechanics. An exact analysis of this problem leads to the Bell inequality:

$$(1)\ 1 + C(b,c) \geq \left| C(a,b) - C(a,c) \right|$$

In this formula a, b and c are the settings of the apparatus in the x-z-plane where Alice and Bob perform their spin measurements. The plus indicates that it should be the probability of getting spin up. This inequality must be fulfilled by all theories postulating local hidden variables. Another more general form of this inequality is the CHSH-inequality[ix][x]:

$$(2)\ -2 \leq E(a,b) - E(a,b') + E(a',b) + E(a',b') \leq +2$$

Due to the fact that Quantum Mechanics and the experiments violate this formula, the Bell's theorem follows:
"No physical theory of local hidden variables can ever reproduce all the predictions of quantum mechanics"
The validity of the inequality has been shown by many successful experiments. For more details to Bell's theorem see the handout of Yvonne Ventura[xi].

Loopholes

These experiments are mainly in two points criticised which are called the "locality loophole" and the "detection loophole". The locality loophole means that it must be ensured that Alice and Bob don't know which measurement the other performs. This loophole can be closed by putting enough distance between Alice and Bob and that both make their decisions which measurement they will perform fast enough, so that the other can't know it due to the by the limited speed of information transfer(light speed). No all photons that reach the detector are really detected and this is called the detection loophole. You must show that the non detected particles have the same features as the detected (fair sampling theorem). But today there are CCD-detectors with a quantum efficiency of up to 99% which can also close this loophole. Beside those loopholes I've already mentioned there are others, which are based on errors of the components of specific experiments. Furthermore the statistical nature of the Bell inequality has also been criticised, but this problem has been solved by the GHZ-experiment[xii].[xiii][xiv] By using 3 entangled particles you can proof the violation of the Bell inequality with only four measurements.

KOCHEN-SPECKER THEOREM

History

1932 John von Neumann published a proof which showed that quantum mechanics and hidden variables aren't combinable if you require non-contextuality. Despite this proof had been cited very often, it was based on wrong assumptions. Based on A.M. Gleason's[xv] work in 1957 and Bell's work in 1966 Simon Kochen and Ernst Specker[xvi] were able to develop a new theorem a year later. By now the Kochen-Specker Theorem has been experimentally verified with several different particles (photons, neutrons) during the last years.

The Kochen-Specker Theorem[xvii]

"There is no non-contextual model with hidden variables in quantum mechanics"
But you can put this statement also into the language of mathematics[xviii]:
Let H be a Hilbert space of quantum mechanical state vectors of dimension $x \geq 3$. There is a set M of observables on H, containing y elements, such that the following two assumptions are contradictory:

- All y members of M simultaneously have values, i.e. are unambiguously mapped onto real numbers (designated, for observables A, B, C, …, by $v(A)$, $v(B)$, $v(C)$, …).
- Values of observables conform to the following constraints:

If A, B, C are all compatible and $C = A+B$, then $v(C) = v(A)+v(B)$;

If A, B, C are all compatible and $C = A \cdot B$, then $v(C) = v(A) \cdot v(B)$.

The heart of statement is much more than a simple theorem about the geometrical structure of the quantum mechanical Hilbert-space. It forbids a certain class of hidden variable theories, that isn't prohibited by Bell's theorem by showing that realism and non-contextuality cause a contradiction. Furthermore some arithmetic rules for measured values of systems of several observables follow out of the Kochen-Specker theorem.

Mathematical Background

I'll try to give you a short overview about the derivation of the Kochen-Specker Theorem. I will stick strongly to David Mermin's[xix] colourful geometric way to do this. A more detailed derivation and proof of the Kochen-Specker theorem can be found their own 29 pages long original paper.

We start to describe a three-dimensional state space in terms of observables built out of angular momentum components of the spin along various directions. The eigenvalues of these observables are either 0 or 1 and the squared spin components along three orthogonal directions u, v and w have to fulfil this equation:

$$(3)\quad S_u^2 + S_v^2 + S_w^2 = s(s+1) = 2$$

Furthermore these squared spin components are mutually commuting due to the fact that we have spin 1 and so they can be simultaneously measured. We are now producing a set of directions for which there is no way whatever to assign 1's and 0's to the directions consistent with formula (3), thereby rendering the statistical state-dependent part of the argument unnecessary. For this aim we have to find a set of three-dimensional vectors that obeys several conditions. The colour red stands for the value "1" to the squared spin component along a direct and the colour blue stands for "0". It shall be impossible to colour each vector red or blue by using a subset containing one blue and two red vectors. Now we have to show that if the angle between two vectors of different colour is less than arctan (0.5), then we can find additional vectors which constitute a set (with the original two vectors) that can't be coloured according to the rules. We define the blue vector as the unit vector of the z-axis "z" and demand of the red vector "a" to be place in the y-z plane:

$$(4)\ a = z + \alpha y, \quad 0 < \alpha < 0.5$$

This assumption supplies us with important observations. The x-y plane must be red and so the x and y vector too. As a consequence the vector

$$(5)\ c = \beta x + y$$

has to be red too. Furthermore another red vector would be:

$$(6)\ d = \frac{x}{\beta} - \frac{a}{\alpha}$$

The normal of the plane of c and d must be blue and every vector in the plane has to red like:

$$(7)\ e = c + d = (\beta + \frac{1}{\beta})x - \frac{z}{\alpha}$$

The reciprocal value of α must be larger than 2 and the absolute value of the sum of β and its own reciprocal value has to be between 2 and infinite. We can find a β where e is along the direction f or along g which both have to be red because e is red.

$$(8)\ \begin{aligned} f &= x - z \\ g &= -x - z \end{aligned}$$

The vectors f and g are orthogonal so their plane is red and its normal vector has to be blue. Now we have our contradiction because z which can be written as a linear combination of f and g is per definition blue. If a and z have different colours, the set can't be coloured according to the rules. The same procedure can be repeated in the y-x plane and so on. Kochen and Specker explicitly displayed a finite set of 117 directions which cannot be coloured, but there are other sets with fewer directions too.

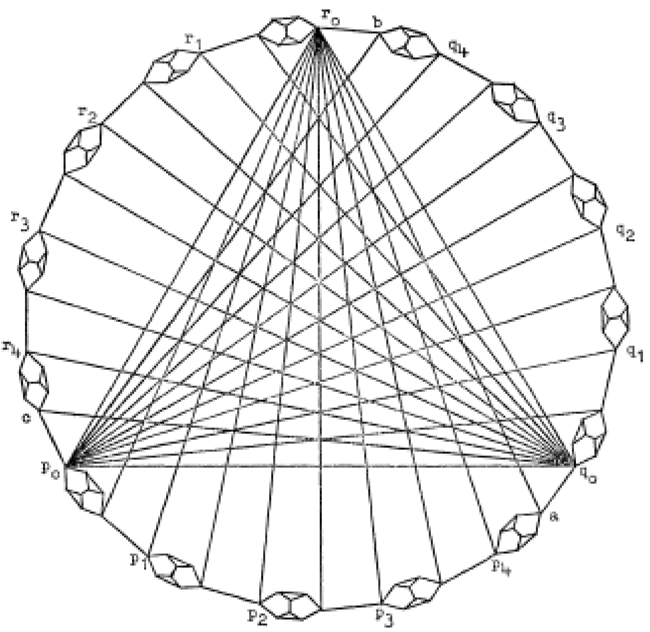

Fig.2: Kochen and Specker used this diagram to represent their set of uncolourable directions.

The Kochen-Specker-Theorem and its proof become simpler in four dimensions. Therefore we represent our observables in terms of the Pauli matrices for two independent spin ½-particles $\sigma^1{}_\mu$ and $\sigma^2{}_\nu$. The squares of the Pauli matrices are unity and so the eigenvalues are ±1. Furthermore we can use many well know properties of the Pauli matrices.

σ_x^1	σ_x^2	$\sigma_x^1\sigma_x^2$
σ_y^2	σ_y^1	$\sigma_y^1\sigma_y^2$
$\sigma_x^1\sigma_y^2$	$\sigma_x^2\sigma_y^1$	$\sigma_z^1\sigma_z^2$

Fig.3: These nine observables are the basis to a proof of the Kochen-Specker theorem in four or more dimensions.

In Fig.3 the observables in each row and each column are mutually commuting. The product of the observables in each row and each column, except the right one where it is -1, is +1.Since the values assigned to mutually commuting observables must obey any identities satisfied by the observables themselves, the identities mentioned in the sentence before require the product of the values assigned to three observables in each row and the first two columns to be +1 and for the last column to be -1. This can't be satisfied, since the row identities require the product of all nine values to be 1, while the column identities require it to be -1. The Kochen-Specker theorem in 8 dimensions is quite similar to the one in four.

CONTEXTUALITY AND LOCALITY

Locality[xx]

Locality means that interactions are limited to the immediate surroundings. There is no action at distance. Distance in this case means that both interacting objects must be in causal contact. An event (in the sense of special relativity) can't influence another event that is outside its light cone[xxi]. So locality ensures causality and forbids information to travel faster than light. Bell's theorem states that hidden variable theories are incompatible with locality.

Contextuality

If the results depend on the context of the experiment, then we call it contextual. In the case of quantum mechanics this means: The result of a measurement of A depends on another measurement on observable B, although these two observables commute with each other. The opposite of contextuality is called non-contextuality. Locality is seen as a special case of non-contextuality, because it requires mutual independence of the results for commuting observables even if there is no spacelike separation. Let's take two sets of mutually commuting variables A, B, C… and A, L, M…. But not all B, C… commute with all L, M…. If the value of A measured with the first set is the same as if you have measured it with the second, you this fact non-contextuality. Due the Kochen-Specker theorem there is no non-contextual hidden variable theory, so if we still want to have hidden variables we must develop a contextual theory. This attempt would leave us with the question why a slightly different arrangement of the measurement of predetermined hidden values would give us different results. Most hidden variable theories can be eliminated by the Kochen Specker theorems and those which don't violate this theorem, like Bohmian mechanics, have serious problems to explain all consequences of their non-locality and contextuality.

EXPERIMENTS

Single Photon/Particle Experiment

This experiment was proposed by Christoph Simon, Marek Zukowski, Harald Weinfurther and Anton Zeilinger[xxii] in 2000 and it was accomplished by Yun-Feng Huang, Chuan-Feng Li, Yong-Sheng Zhang, Jian-Wei Pan and Guang-Can Guo[xxiii] in 2002. In this experiment the spin/polarisation of the particle and its path are used as degrees of freedom, which gives us a non-statistical test of non-contextual hidden variables versus quantum mechanics. Let's take 4 observables Z_1, X_1, Z_2, X_2 and each can give us two possible results: ±1. Furthermore they have predetermined non-contextual values ±1 for individual systems: $v(Z_1)$, $v(X_1)$, $v(Z_2)$, $v(X_2)$. The result of a measurement of Z_1 will always be $v(Z_1)$ for an individual system. It doesn't matter which other observables are measured with it, but this leads us to a contradiction between non-contextuality and quantum mechanics. We are now using an ensemble E for which we can always find the same results for Z_1 and Z_2 and also for X_1 and X_2. For each system of that ensemble follows:

$$(9)\quad v(Z_1) = v(Z_2) \qquad and \qquad v(X_1) = v(X_2)$$

In addition to that $v(Z_1)$ can either be the same as $v(X_1)$ or not and this leads us after a few steps to a contradiction between non-contextuality and quantum mechanics.

$$(10)\quad \begin{array}{l} v(Z_1).v(Z_2) = v(X_1).v(X_2) = 1 \\ \Rightarrow v(Z_1).v(X_2) = v(X_1).v(Z_2) \end{array}$$

To measure a product of observables you can measure both separately and multiply the result. This works in a non-contextual theory but not in general. In case of non-contextuality we get:

$$(11)\quad v(Z_1.X_2) = v(Z_1).v(X_2)$$

We can apply this result on equations (10). Beside non-contextuality we must ensure that Z_1X_2 and X_1Z_2 are comeasurable. Now we are considering a quantum mechanical system of two qubits and the observables:

$$(12)\quad \begin{matrix} Z_1 = \sigma_Z^{(1)} & X_1 = \sigma_X^{(1)} \\ Z_2 = \sigma_Z^{(2)} & X_2 = \sigma_X^{(2)} \end{matrix}$$

A joint eigenstate of the commuting product observables Z_1Z_2 and X_1X_2 with both eigenvalues equal to +1 would be this two-qubit state:

$$(13)\quad |\psi_1\rangle = \frac{1}{\sqrt{2}}(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle = \frac{1}{\sqrt{2}}(|+x\rangle|+x\rangle + |-x\rangle|-x\rangle$$

Quantum mechanics predicts for that stat that the measured value of Z_1X_2 will always be opposite to the value of X_1Z_2. This can be seen here:

$$|\chi_{1,-1}\rangle \equiv \frac{1}{2}(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle + |+z\rangle|-z\rangle - |-z\rangle|+z\rangle) = \frac{1}{\sqrt{2}}(|+z\rangle|+x\rangle - |-z\rangle|-x\rangle) = \frac{1}{\sqrt{2}}(|-x\rangle|+z\rangle + |+x\rangle|-z\rangle)$$

$$|\chi_{-1,1}\rangle \equiv \frac{1}{2}(|+z\rangle|+z\rangle + |-z\rangle|-z\rangle - |+z\rangle|-z\rangle + |-z\rangle|+z\rangle) = \frac{1}{\sqrt{2}}(|+z\rangle|-x\rangle + |-z\rangle|+x\rangle) = \frac{1}{\sqrt{2}}(|+x\rangle|+z\rangle - |-x\rangle|-z\rangle)$$

$$(14)\quad |\psi_1\rangle = \frac{1}{\sqrt{2}}(|\chi_{1,-1}\rangle + |\chi_{-1,1}\rangle)$$

$$Z_1X_2|\chi_{1,-1}\rangle = +|\chi_{1,-1}\rangle \qquad X_1Z_2|\chi_{1,-1}\rangle = -|\chi_{1,-1}\rangle$$

$$Z_1X_2|\chi_{-1,1}\rangle = -|\chi_{-1,1}\rangle \qquad X_1Z_2|\chi_{-1,1}\rangle = +|\chi_{-1,1}\rangle$$

From these equations follows that in a joint measurement of the two observables Z_1X_2 and X_1Z_2 they will always be found different. As a consequence the ensemble ψ_1 can't be described by any non-contextual theory. The experiment can be performed with a single spin-½ particle or a photon (using polarisation instead of spin) as it was done. To measure your states you only need a source of single particles, beam splitters and Stern-Gerlach-type[xxiv] devices (consist of two magnets different shaped that produce an inhomogeneous magnetic field). Our state ψ_1 can be mapped onto the one-particle state:

$$(15)\quad |\psi_1\rangle = \frac{1}{\sqrt{2}}(|u\rangle|z+\rangle + |d\rangle|z-\rangle)$$

if u and d are two spatial modes and z+ and z- are the spin states before them. Our 4 observables are therefore:

$$Z_1 = |u\rangle\langle u| - |d\rangle\langle d| \qquad X_1 = |u'\rangle\langle u'| - |d'\rangle\langle d'|$$
$$(16)\quad Z_2 = |z+\rangle\langle z+| - |z-\rangle\langle z-| \qquad X_2 = |x+\rangle\langle x+| - |x-\rangle\langle x-|$$

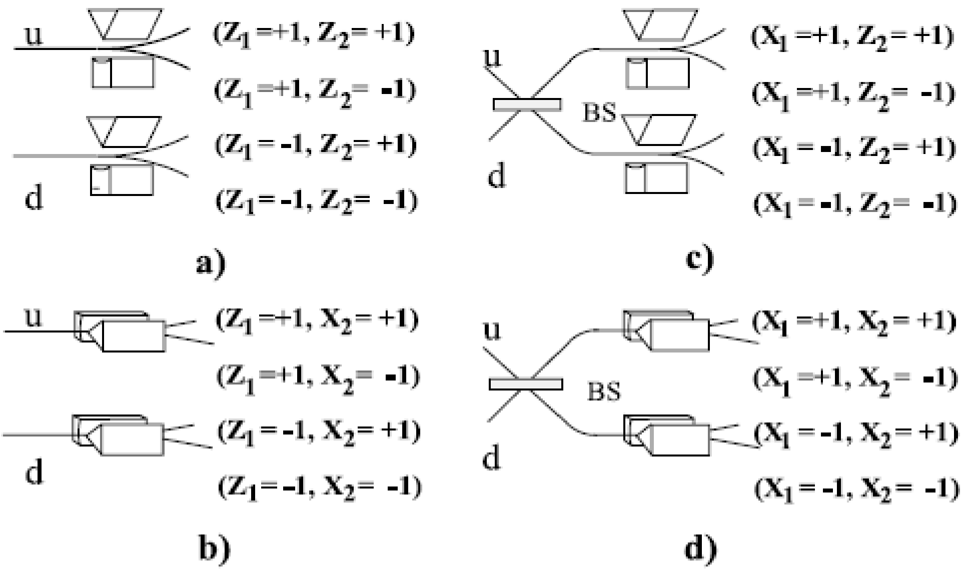

Fig.4: devices for measuring pairs of the single-particle observables due to the idea of Simon et al

When Huang et al performed their experiment they used single photon detectors (D0-D8) with an efficiency of ~70% at 702.2nm, polarizing beam splitters (PBS), rotated half-wave plates (HWP) at a special angle and “normal” beam splitters (BS). Their single photon was one photon of a photon pair produced in process in a beta-barium-borate crystal.

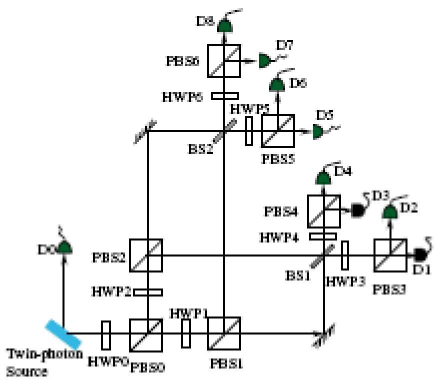

Fig.5: the experimental setup of Huang et al

The results of their measurement showed a clear agreement with quantum mechanics and only about 20% of the results have been in agreement with local hidden variables (see Fig.6).

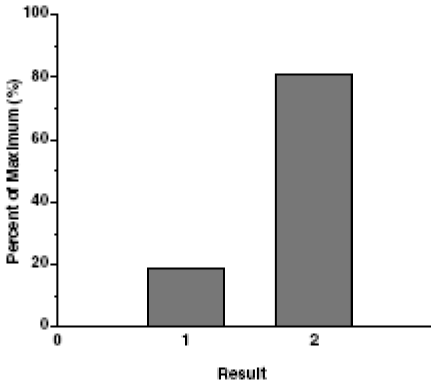

Fig.6: Result 1 is the fraction of total coincidence rates that agrees with non-contextual hidden variables and result 2 is the fraction that agrees with quantum mechanics.

Neutron Optical Experiment

In 2003 Yuji Hasegawa, Rudolf Loidl, Gerald Badurek, Matthias Baron and Helmut Rauch[xxv] performed a single-neutron optical experiment to proof the validity of the Kochen-Specker theorem. In this experiment they prepared a neutron in a non-factorizable state and made a joint measurement of commuting observables of this particle.

$$(17)\quad |\psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle \otimes |p_1\rangle + |\uparrow\rangle \otimes |p_2\rangle)$$

This (17) is the normalized total wavefunction. The ket-vectors p_1 and p_2 represent the two possible beam paths in the interferometer and the others are the spin states. This wavefunction resembles the one (15) of the previous experiment. Now we will introduce in each case two operator projecting the spin or the path part into orthogonal states.

$$(18)\quad \begin{aligned} P^s_{\alpha;\pm 1} &= \frac{1}{2}(|\uparrow\rangle \pm e^{i\alpha}|\downarrow\rangle)(\langle\uparrow| \pm e^{-i\alpha}\langle\downarrow|) \\ P^p_{\chi;\pm 1} &= \frac{1}{2}(|p_1\rangle \pm e^{i\chi}|p_2\rangle)(\langle p_1| \pm e^{-i\chi}\langle p_2|) \end{aligned}$$

We use this to calculate our expectation value for a joint measurement of a spin and a path state.

$$(19)\quad E(\alpha,\chi)=\langle\psi|P^s(\alpha)P^p(\chi)|\psi\rangle=\langle\psi|[(+1)P^s{}_{\alpha;+1}+(-1)P^s{}_{\alpha;-1}].[(+1)P^p{}_{\chi;+1}+(-1)P^p{}_{\chi;-1}]|\psi\rangle$$

Both observables (spin and path) of these projection operators operate in different Hilbert spaces and thus they commute. By using the CHSH inequality (see equation (2)) we get following condition:

$$(20)\quad \begin{array}{c} -2\leq S\leq +2 \\ S=E(\alpha_1,\chi_1)-E(\alpha_1,\chi_2)+E(\alpha_2,\chi_1)+E(\alpha_2,\chi_2) \end{array}$$

The projection operators above can be realised in the experiment by a spin rotator for α and a phase shifter for χ. By comparing coincidence counts and using some quantum theory you can derive the behaviour of the expectation value:

$$(21)\quad E(\alpha,\chi)=\cos(\alpha+\chi)$$

Now you can find angles where the maximum violation of equation (20) is expected and there the value of S is around 2.82 and therefore greater than 2.

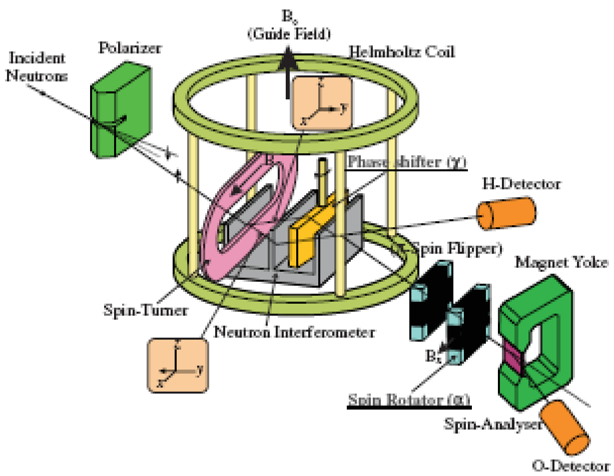

Fig.7: the experimental setup of Hasegawa et al

The neutrons used in this experiment were monochromatized to a mean wavelength pf 1,92Å by using a silicon perfect crystal monochromator. The polarized beam was into two paths to produce a Bell state. A uniform magnetic field along the z-axis was provided by a Helmholtz Coil. The beam was sent into a neutron interferometer where a spin-tuner and a phase shifter had been placed. After this one beam had to pass a spin rotator and a spin analyser before reaching the O-detector, which has an efficiency of more than 99%. During the experiment the parameters α and χ had been varied to achieve sufficiently high contrast values. Despite the highly efficient detectors a fair sampling theorem (see loopholes) is required because of the losses in the interferometer. The result of S=2.051±0.019 shows a violation of inequality (20), which has been derived in this case from contextuality. Therefore the Kochen-Specker theorem has been shown to be valid once again.

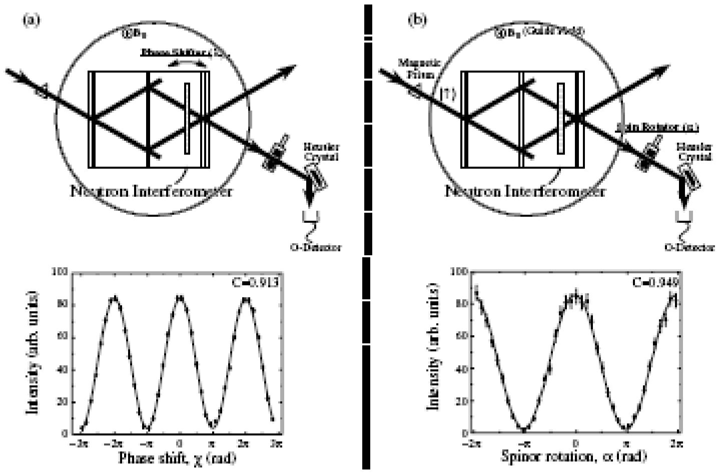

Fig.8: interference oscillations of path and spin shows the capability of the experimental setup to manipulate the path and spinor subsystems

CONCLUSION

The Kochen-Specker Theorem, although it is less famous than Bell's Inequality, has shown to be more powerful to falsify a certain important class of hidden-variable theories than it. This gives us strong arguments in favour of quantum mechanics and the Copenhagen interpretation. There are still possible interesting hidden-variable theories left, like Bohmian mechanics, they have significant problems to explain why contextuality is necessary for them. It is comparable to the debate about Big-Bang versus Steady-State. In both theories it is possible to explain phenomena like the comical microwave background (CMB), but in one it is much less complicated and clearer. Locality isn't required for a non-relativistic theory like Bohmian mechanics, but all attempts to make it more general have failed up to now. Under this conditions and the good experimental conformation of the Kochen-Specker Theorem it is no longer reasonable to keep hidden-variable theories and accept that the universe isn't deterministic in the classical sense. But on the other hand remember Einstein: "Everything should be made as simple as possible, **but not simpler**."[xxvi]

REFERENCES

[i] http://en.wikipedia.org/wiki/Copenhagen_interpretation (17. 12. 2007)
[ii] Friis N.: Interpretationen der Quantentheorie am Beispiel der Verschränkung; Grundlagen der Modernen Quantenphysik – Seminar; 2006
[iii] Bohm D.: A Suggested Interpretation of the Quantum Theory in terms of "Hidden" Variables; Physical Review 82-2; 1952
[iv] http://en.wikipedia.org/wiki/Interpretation_of_quantum_mechanics (17. 12. 2007)
[v] http://en.wikipedia.org/wiki/EPR_paradox (17.12. 2007)
[vi] Bell J.: On the Problem of Hidden Variables in Quantum Mechanics; Review of Modern Physics 38-3; 1966
[vii] Peres A.: Unperformed experiments have no results; American Journal of Physics 46; 1978
[viii] http://en.wikipedia.org/wiki/Bell%27s_theorem (17. 12. 2007)
[ix] Clausner J., Horne M., Shimony A. and Holt R.: Proposed experiment to test local hidden-variable theories; Physical Review Letters 23-15; 1969
[x] http://en.wikipedia.org/wiki/CHSH_inequality (17. 12. 2007)
[xi] Ventura Y.: Entanglement and Bell Theorem; Grundlagen der Modernen Quantenphysik – Seminar; 2007
[xii] Greenberger D., Horne M. and Zeilinger A.: Going beyond Bell's Theorem; 1989
[xiii] Greenberger D., Horne M., Shimony A. and Zeilinger A.: Bell's theorem without inequalities; American Journal of Physics 58;1990
[xiv] see also: Kotyrba M.: GHZ; Grundlagen der Modernen Quantenphysik - Seminar; 2007
[xv] Gleason A.: Measures on the Closed Subspaces of a Hilbert Space; Journal of Mathematics and Mechanics 6-4; 1957
[xvi] Kochen S. and Specker E.: The Problem of Hidden Variables in Quantum Mechanics; Journal of Mathematics and Mechanics 19-1; 1967
[xvii] http://de.wikipedia.org/wiki/Kochen-Specker-Theorem (17. 12. 2007)
[xviii] http://plato.stanford.edu/entries/kochen-specker/ (17. 12. 2007)
[xix] Mermin N.: Hidden variables and the two theorems of John Bell; Review of Modern Physics 65-3; 1993
[xx] http://en.wikipedia.org/wiki/Principle_of_locality (17. 12. 2007)
[xxi] http://en.wikipedia.org/wiki/Light_cone (17. 12. 2007)
[xxii] Simon C., Zukowski M., Weinfurther H. and Zeilinger A.: Feasible „Kochen-Specker" Experiment with Single Particles; The American Physical Society 85-9; 2000
[xxiii] Huang Y., Li C., Zhang Y., Pan J. and Guo G.: Experimental Test of The Kochen-Specker Theorem with Single Photons; The American Physical Society 90-25; 2003
[xxiv] http://en.wikipedia.org/wiki/Stern%E2%80%93Gerlach_experiment (17. 12. 2007)
[xxv] Hasegawa Y., Loidl R., Badurek G., Baron M. and Rauch H.: Violation of a Bell-like inequality in neutron optical experiments: quantum contextuality; Journal of Optics B: Quantum and Semiclassical Optics 6; 2004
[xxvi] http://en.wikiquote.org/wiki/Albert_Einstein (17. 12. 2007)